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## FADDEEV-TYPE CALCULATIONS OF FEW-BODY NUCLEAR REACTIONS INCLUDING COULOMB INTERACTION

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The method of screening and renormalization is used to include the Coulomb interaction between the charged particles in the description of few-body nuclear reactions. Calculations are done in the framework of Faddeev-type equations in momentum-space. The reliability of the method is demonstrated. The Coulomb effect on observables is discussed.

*Keywords:* Screening and renormalization; few-body scattering; Faddeev equations.

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### 1. Introduction

The inclusion of the Coulomb interaction in the description of the three-particle scattering is a challenging task in theoretical few-body nuclear physics. Due to its long range, the Coulomb potential  $w(r)$  does not satisfy the mathematical properties required for the formulation of the standard scattering theory. There is a number of suggestions how to overcome this difficulty; most of them are based on the configuration-space framework<sup>1,2,3,4</sup> and are limited to energies below three-body breakup threshold (3BBT), while the others<sup>5,6,7</sup> have not matured yet into practical applications. Up to now only few approaches led to the results above 3BBT. Those are configuration-space calculations for proton-deuteron ( $p$ - $d$ ) elastic scattering using the Kohn variational principle<sup>8</sup> and the screening and renormalization approach in the framework of momentum-space integral equations;<sup>9,10,11,12</sup> the latter method will be discussed in more details. Very recently  $p$ - $d$  results above 3BBT were also obtained using modified Faddeev equation in configuration space together with the dumping (screening) of particular Coulomb contributions.<sup>13</sup>

### 2. Method of screening and renormalization

Our treatment of the Coulomb interaction is based on the idea of screening and renormalization proposed in Refs. 14, 15 for the scattering of two charged particles. The standard scattering theory is formally applicable to the screened Coulomb

potential which, in the  $r$ -space representation, we choose as

$$w_R(r) = w(r) e^{-(r/R)^n}, \quad (1)$$

where  $R$  is the screening radius, and  $n$  controls the smoothness of the screening. In 1974 Taylor<sup>14</sup> suggested that even for the description of systems with the Coulomb interaction the standard scattering theory may be useful, since in nature the Coulomb potential is always screened. The study of the two-particle system with the screened Coulomb interaction revealed that, as expected, the physical observables become insensitive to screening provided it takes place at sufficiently large distances  $R$  and, in the  $R \rightarrow \infty$  limit, coincide with the corresponding quantities known from the analytical solution of the two-particle Coulomb problem: though the on-shell screened Coulomb transition matrix  $\langle \mathbf{p}_f | t_R(e_i + i0) | \mathbf{p}_i \rangle$  with energy  $e_i$  and momenta  $p_f = p_i$  diverges in the  $R \rightarrow \infty$  limit, after renormalization by (an equally) diverging phase factor  $z_R^{-1}(p_i)$  it converges as a distribution to the well known proper Coulomb amplitude  $\langle \mathbf{p}_f | t_C | \mathbf{p}_i \rangle$ , i.e.,

$$\lim_{R \rightarrow \infty} \langle \mathbf{p}_f | t_R(e_i + i0) | \mathbf{p}_i \rangle z_R^{-1}(p_i) = \langle \mathbf{p}_f | t_C | \mathbf{p}_i \rangle. \quad (2)$$

Renormalization by  $z_R^{-\frac{1}{2}}(p_i)$  in the  $R \rightarrow \infty$  limit relates also the screened and proper Coulomb wave functions.<sup>16</sup> The convergence of the scattering amplitude as a distribution corresponds to the physical conditions in a real experiment where the incoming beam is not a plane wave but a wave packet and the forward scattering cannot be measured.<sup>14</sup> This justifies the replacement (2) in practical calculations. We emphasize that the renormalization (2) relates only the scattering amplitudes; its application to the off-shell transition matrices<sup>17</sup> is unjustified. However, for the calculation of observables, only the renormalization of the on-shell transition matrix and wave function is needed.

The screening and renormalization approach can be applied to more complicated systems.<sup>18</sup> Here we briefly recall the procedure which is described in details in Refs. 11, 12. In the multichannel on-shell transition matrix  $\langle f | T_{\beta\alpha}^{(R)}(E_i + i0) | i \rangle$  between initial and final channel states  $|i\rangle$  and  $|f\rangle$ ,  $E_f = E_i$ , derived from nuclear plus screened Coulomb potentials, one has to isolate the diverging screened Coulomb contributions in the form of a two-body on-shell transition matrix and two-body wave function with known renormalization properties. This can be achieved using the two-potential formalism as long as in the initial/final states there are no more than two charged bodies (clusters). This also enables to decompose  $\langle f | T_{\beta\alpha}^{(R)}(E_i + i0) | i \rangle$  into contributions with different range properties. The long-range part, the two-body on-shell transition matrix  $\langle f | T_{\alpha R}^{c.m.}(E_i + i0) | i \rangle$ , derived from the screened Coulomb potential of the form (1) between the centers of mass (c.m.) of the two charged bodies in the initial state, is present in the elastic scattering only. After renormalization by the diverging phase factor  $Z_{iR}^{-1}$ , this contribution converges towards its  $R \rightarrow \infty$  limit very slowly, and, in general, as a distribution only, but the result  $\langle f | T_{\alpha C}^{c.m.} | i \rangle$ , the pure Coulomb amplitude of two-body nature, is known analytically.

The remaining part of the elastic scattering amplitude as well as the amplitudes for transfer and breakup are short-range operators that are externally distorted by Coulomb. Due to their short-range nature, convergence with  $R$  after the renormalization by the corresponding phase factors is fast and, therefore, the  $R \rightarrow \infty$  limit can be calculated numerically with high accuracy at finite  $R$ . Thus, the physical scattering amplitudes are obtained after renormalization of  $\langle f | T_{\beta\alpha}^{(R)}(E_i + i0) | i \rangle$  in the  $R \rightarrow \infty$  limit as

$$\begin{aligned} \langle f | T_{\beta\alpha} | i \rangle &= \delta_{\beta\alpha} \langle f | T_{\alpha C}^{c.m.} | i \rangle \\ &+ \lim_{R \rightarrow \infty} \{ Z_{fR}^{-\frac{1}{2}} \langle f | [T_{\beta\alpha}^{(R)}(E_i + i0) - \delta_{\beta\alpha} T_{\alpha R}^{c.m.}(E_i + i0)] | i \rangle Z_{iR}^{-\frac{1}{2}} \}. \end{aligned} \quad (3)$$

One can use standard scattering theory to calculate the multichannel transition operators  $T_{\beta\alpha}^{(R)}(E_i + i0)$  at finite screening radius  $R$  and make sure that  $R$  is large enough for the convergence of the results. We solve Alt, Grassberger, and Sandhas (AGS) equations for three- and four-particle scattering<sup>19,20</sup> which are equivalent to Faddeev and Yakubovsky equations.<sup>21,22</sup> We employ momentum-space partial-wave representation as described in detail in Refs. 23, 24, 25 for three- and four-nucleon scattering without the Coulomb force. However, the screened Coulomb interaction, due to its longer range, compared to the nuclear interaction, brings additional difficulties: quasisingular nature of the potential and slow convergence of the partial-wave expansion. The right choice of the parameter  $n$  controlling the smoothness of the screening (1) is essential in resolving those difficulties as we demonstrated in Ref. 11. We work with a sharper screening than the Yukawa screening ( $n = 1$ ) of Refs. 9, 10. We want to ensure that the screened Coulomb potential  $w_R(r)$  approximates well the true Coulomb one  $w(r)$  for distances  $r < R$  and simultaneously vanishes rapidly for  $r > R$ , providing a comparatively fast convergence of the partial-wave expansion and less pronounced quasisingularities. In contrast, the sharp cutoff ( $n \rightarrow \infty$ ) yields an unpleasant oscillatory behavior in the momentum-space representation, leading to convergence problems. Depending on the reaction we found the values  $3 \leq n \leq 8$  to provide a sufficiently smooth, but at the same time a sufficiently rapid screening around  $r = R$ . In any case the screening radius  $R$  needed for convergence in Eq. (3) is considerably larger than the range of the nuclear interaction and, therefore, the calculation of  $T_{\beta\alpha}^{(R)}(E_i + i0)$  requires the inclusion of partial waves with angular momentum much higher than needed for the nuclear potential alone. This problem can be solved in an efficient and reliable way either by using the perturbative approach for high two-particle partial waves, developed in Ref. 26, or even without it as discussed in Ref. 27.

The internal criterion for the reliability of our method is the convergence of the observables with screening radius  $R$  used to calculate the Coulomb-distorted short-range part of the amplitudes in Eq. (3). Numerous examples for three-nucleon hadronic and electromagnetic reactions,  $\alpha$ - $d$  and  $p$ - $^3\text{He}$  scattering can be found in Refs. 11, 12, 28, 29, 30. In most cases the convergence is impressively fast; the screening radius  $R = 10$  to 30 fm is sufficient. The exceptions requiring larger screening

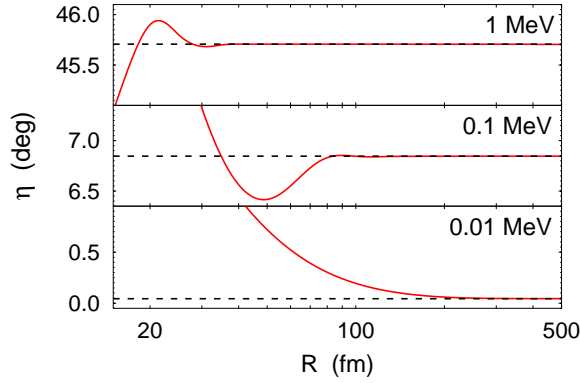


Fig. 1.  $^1S_0$  phase shifts of  $pp$  scattering at 0.01, 0.1 and 1 MeV c.m. energy. Results obtained using screening and renormalization method are shown as function of the screening radius  $R$  (solid curves). Dashed lines are the exact results.

radii are the observables at very low energies and the breakup differential cross section in kinematical situations characterized by very low relative energy  $E_{\text{rel}}$  between the two charged particles, e.g.,  $p$ - $d$  breakup or photodisintegration of  $^3\text{He}$  close to the  $pp$  final-state interaction ( $pp$ -FSI) regime.<sup>12</sup> In there, the Coulomb repulsion is responsible for decreasing the cross section, converting the FSI peak obtained in the absence of Coulomb into a minimum with zero cross section at  $E_{\text{rel}} = 0$ . Such a behavior is seen in the experimental data as well.<sup>31,32</sup> The slow convergence under those conditions is not surprising, since the renormalization factor itself as well as the Coulomb parameter become ill-defined, indicating that the screening and renormalization procedure cannot be applied at  $E_{\text{rel}} = 0$ . Therefore an extrapolation has to be used to calculate the observables at  $E_{\text{rel}} = 0$ , which works pretty well since the observables vary smoothly with  $E_{\text{rel}}$ .

It has been claimed<sup>17</sup> that the method of screening and renormalization ceases to work even for  $pp$  scattering below 0.1 MeV c.m. energy. In Fig. 1 we prove that this is not so by getting well converged results at 0.01 MeV. However, Fig. 1 also shows that the screening radius indeed has to be increased with decreased energy and at some point would become too large for reliable numerical calculation.

### 3. Results and summary

We have discussed how the Coulomb interaction between the charged particles can be included into the description of few-body reactions using the old idea of screening and renormalization.<sup>14</sup> The calculations are done in the framework of AGS integral equations<sup>19,20</sup> in momentum-space. The screening and renormalization approach has already been used for  $p$ - $d$  scattering<sup>9,10,33</sup> but with limited success: those calculations were based on quasiparticle equations with rank-1 separable potentials and, in addition, the screened Coulomb transition matrix was approximated by the

screened Coulomb potential; none of these approximations is used by us. It is the new screening function that allows us to avoid these approximations and obtain fully converged results. Furthermore, the results for  $p$ - $d$  elastic scattering obtained by the present technique were compared<sup>34</sup> with those of Ref. 8 obtained from the variational solution of the three-nucleon Schrödinger equation in configuration space with the inclusion of an *unscreened* Coulomb potential between the protons and imposing the proper Coulomb boundary conditions explicitly. Good agreement over a wide energy range was found indicating that both techniques for including the Coulomb interaction are reliable. At very low energies the coordinate-space treatments remain favored since there the method of screening and renormalization converges slowly and therefore becomes technically too demanding, but at higher energies and for three-body breakup reactions it is more efficient. A detailed comparison still has to be done but it seems that there is a reasonable agreement between momentum- and coordinate-space results also in the case of  $p$ - $d$  breakup<sup>13</sup> and  $p$ - $^3\text{He}$  scattering.<sup>35</sup>

The present method was used to study  $p$ - $d$  elastic scattering and breakup in Refs. 11, 12, 31, 32, 36,  $p$ - $d$  radiative capture in Refs. 11, 37 and photo- and electrodisintegration of  $^3\text{He}$  in Refs. 11, 12, 38. Furthermore, it was applied to the nuclear reactions dominated by three-body degrees of freedom, i.e., deuteron scattering on stable nuclei or proton scattering on one-neutron halo nuclei. Examples are low-energy  $\alpha$ - $d$  elastic scattering and breakup<sup>28</sup> and  $d + ^{12}\text{C}$  and  $p + ^{11}\text{Be}$  elastic, transfer, and breakup reactions<sup>39,40,41</sup> where also the accuracy of traditional approximate nuclear reaction approaches like Continuum Discretized Coupled Channels (CDCC) method, Glauber, and Distorted Wave Impulse Approximation (DWIA) could be tested. Finally, all elastic and transfer four-nucleon reactions below three-body breakup threshold, i.e.,  $n + ^3\text{H}$ ,  $p + ^3\text{He}$ ,  $p + ^3\text{H}$ ,  $n + ^3\text{He}$ , and  $d + d$ , have been studied in Refs. 25, 30, 42, 43. The conclusion is that the Coulomb effect is important at low energies for all kinematic regimes, but gets confined to the forward direction in elastic scattering at higher energies. In  $p$ - $d$  breakup and in three-body e.m. disintegration of  $^3\text{He}$  the Coulomb effect is extremely important in kinematical regimes close to  $pp$ -FSI. There the  $pp$  repulsion converts the  $pp$ -FSI peak obtained in the absence of Coulomb into a minimum with zero cross section.<sup>32</sup> This significant change of the cross section behavior has important consequences in nearby configurations where one may observe instead an increase of the cross section due to Coulomb.<sup>31</sup> However, some of the long-standing discrepancies between experiment and theory like the space star anomaly in  $p$ - $d$  breakup are not resolved by the inclusion of the Coulomb interaction.<sup>32,44</sup> A very strong Coulomb effect is found in  $\alpha$ - $d$  breakup where the shift of  $\alpha p$   $P$ -wave resonance position leads to the corresponding shifts of the differential cross section peaks.<sup>28</sup>

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